MAT 2384-Practice Problems on Numerical Methods for Differential Equations

Question 1. For each of the Following IVP's, apply **Euler Method** with the given step size h to estimate solutions on the given interval. Round your answers to 6 decimal places. Then Solve the IVP exactly and compare your estimations with the Exact values.

1. $y' + 5x^4y^2 = 0$, $y(0) = 1$, $h = 0.1$ on $[0,\ 0.5]$
2. $y' = \frac{1}{2}\pi\sqrt{1-y^2}$, $y(0) = 0$, $h = 0.1$ on $[0,\ 0.3]$
3. $y' = (y+x)^2$, $y(0) = 0$, $h = 0.1$ on $[0,\ 0.4]$

Question 2. For each of the Following IVP's, apply the **Improved Euler Method** with the given step size h to estimate solutions on the given interval. Round your answers to 6 decimal places. Then Solve the IVP exactly and compare your estimations with the Exact values.

1. $y' = y - y^2 = 0$, $y(0) = 0.5$, $h = 0.1$ on $[0,\ 0.3]$
2. $y' + 2xy^2 = 0$, $y(0) = 1$, $h = 0.2$ on $[0,\ 0.6]$
3. $y' = 2(y^2+1)$, $y(0) = 0$, $h = 0.05$ on $[0,\ 0.2]$

Question 3. Consider the IVP:

$$y' = 2x^{-1}\sqrt{y - \ln x} + x^{-1}, \quad y(1) = 0.$$

(1) Verify that the Exact solution is $y = (\ln x)^2 + \ln x$

(2) Use the **Improved Euler Method** to estimate solutions of the IVP on the interval $1 \le x \le 1.6$ using a step size of $h = 0.2$. Round your answers to 6 decimal places

(3) Use the **Runge-Kutta method of order 4** to estimate solutions of the IVP on the interval $1 \le x \le 1.6$ using a step size of $h = 0.2$. Round your answers to 6 decimal places

(4) Make a table to compare your estimates in parts (2) and (3) with the exact values (from part (1)).

Question 4. Use the **Runge-Kutta method of order 4** to estimate solutions of the IVP

$$y' = xy + \cos x, \quad y(0) = 0$$

on the interval $0 \leq x \leq 0.6$ using a step size of $h = 0.2$.